

On the Eigenvalue Problem for a Particular Class of Jacobi Matrices

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A function F with simple and nice algebraic properties is defined on a subset of the space of complex sequences. Some special functions are expressible in terms of F , first of all the Bessel functions of the first kind. A compact formula in terms of the function F is given for the determinant of a Jacobi matrix. Further we focus on the particular class of Jacobi matrices whose parallels to the diagonal are constant and whose diagonal depends linearly on the index. A formula in terms of the function F is derived for the characteristic function. A special basis is constructed in which the Jacobi matrix becomes a sum of a diagonal matrix and a rank-one matrix operator. A vector-valued function on the complex plane is constructed having the property that its values on spectral points of the Jacobi matrix are equal to corresponding eigenvectors. It is shown the spectrum of the infinite Jacobi matrix with linear diagonal and constant parallels coincides with zeros of the Bessel function of the first kind as function of its order.